Appendix: TTT4120 Table of formulas, 2018

A. Sequences:

$\sum_{n=0}^{N-1} \alpha^n = \frac{1-\alpha^N}{1-\alpha}$

$|\alpha| < 1 \Rightarrow \sum_{n=0}^{\infty} \alpha^n = \frac{1}{1-\alpha}$ and $-\sum_{n=-1}^{-\infty} \alpha^n = \frac{1}{1-\alpha}$

$\sum_{n=0}^{N-1} (n+1)\alpha^n = \frac{1-\alpha^N}{(1-\alpha)^2} - \frac{N\alpha^N}{1-\alpha}$; $\alpha \neq 1$

$|\alpha| < 1 \Rightarrow \sum_{n=0}^{\infty} (n+1)\alpha^n = \frac{1}{(1-\alpha)^2}$

B. Linear convolution:

$y[n] = h[n] * x[n] = \sum_{k=-\infty}^{\infty} h[k]x[n-k] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$

$Y(z) = H(z)X(z)$

$Y(f) = H(f)X(f)$

$Y(k) = H(k)X(k), k = 0,1,\dots,N-1$ where $Y(k) = Y(f_k)$ with $f_k = k/N$

C. Transforms:

Z-transform: $H(z) = \sum_{n=-\infty}^{\infty} h[n]z^{-n}$

DTFT: $H(f) = \sum_{n=-\infty}^{\infty} h[n]e^{-j2\pi fn}$

DFT: $H(k) = \sum_{n=0}^{N-1} h[n]e^{-j2\pi nk/N}$ $\quad k = 0,1,\dots,N-1$

IDFT: $h[n] = \frac{1}{N}\sum_{k=0}^{N-1} H(k)e^{j2\pi nk/N}$ $\quad n = 0,1,\dots,N-1$

D. Sampling theorem:

Given an analog signal $x_a(t)$ sampled at $F_s = 1/T$. The DTFT of the resulting discrete-time sequence $x[n] = x_a(t)|_{t=nT}$ is given by

$$X(f) = X(F/Fs) = F_s \sum_{k=-\infty}^{\infty} X_a([f-k]F_s)$$

E. Autocorrelation, energy spectrum and Parseval:

Given a sequence $h[n]$ with finite energy E_h

Autocorrelation: $r_{hh}[l] = \sum_{n=-\infty}^{\infty} h[n]h[n+l]$ $\quad l \in \mathbb{Z}$

Energy spectrum: $S_{hh}(z) = H(z)H(z^{-1}) \Rightarrow S_{hh}(f) = |H(f)|^2$

Parseval's theorem: $E_h = r_{hh}[0] = \sum_{n=-\infty}^{\infty} h^2[n] = \int_0^1 |H(f)|^2 df$

F. Multirate:

Decimation (downsampling) where $T_y = DT_x$

$v(mT_y) = \sum_{k=-\infty}^{\infty} h[(mD-k)T_x]x(kT_x) \quad m \in \mathbb{Z}$

Interpolation (upsampling) where $T_y = T_x/I$

$y(lT_y) = \sum_{n=-\infty}^{\infty} h\left[(l-nI)T_y\right]x(nT_x) \quad l \in \mathbb{Z}$

Rate conversion where $T_y = DT_v = \frac{D}{I}T_x$

$y(lT_y) = \sum_{m=-\infty}^{\infty} h[(lD-mI)T_v]x(mT_x) \quad l \in \mathbb{Z}$

G. Autocorrelation, power density spectrum and Wiener-Khintchin:

Given a wide-sense stationary and ergodic sequence $X[n]$ with infinite energy

Autocorrelation: $\gamma_{XX}[l] = E\{X[n]X[n+l]\} \quad l \in \mathbb{Z}$

Power spectrum: $\Gamma_{XX}(z) = \mathcal{Z}\{\gamma_{XX}[l]\} \Rightarrow$

Wiener-Khintchin: $\Gamma_{XX}(f) = \text{DTFT}\{\gamma_{XX}[l]\} = \sum_{l=-\infty}^{\infty} \gamma_{XX}[l]e^{-j2\pi fl}$

H. Yule-Walker and Normal equations where $a_0 = 1$:

Autocorrelation: $\sum_{k=0}^{P} a_k \gamma_{XX}[n-k] = \sigma_f^2 \delta[n] \quad n = 0, \dots, p$

Normal equations: $\sum_{k=1}^{P} a_k \gamma_{XX}[n-k] = -\gamma_{XX}[n] \quad n = 1, \dots, p$

I. Some common z-transform pairs:

Sequence	Transform	ROC
$\delta[n]$	1	$\forall z$
$a^n u[n]$	$\frac{1}{1-az^{-1}}$	$\|z\| > \|a\|$
$-b^n u[-n-1]$	$\frac{1}{1-bz^{-1}}$	$\|z\| < \|b\|$
$(a^n \sin\omega_0 n)u[n]$	$\frac{(a\sin\omega_0)z^{-1}}{1-(2a\cos\omega_0)z^{-1}+a^2z^{-2}}$	$\|z\| > \|a\|$
$(a^n \cos\omega_0 n)u[n]$	$\frac{1-(a\cos\omega_0)z^{-1}}{1-(2a\cos\omega_0)z^{-1}+a^2z^{-2}}$	$\|z\| > \|a\|$
$na^n u[n]$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\|z\| > \|a\|$
$-nb^n u[-n-1]$	$\frac{bz^{-1}}{(1-bz^{-1})^2}$	$\|z\| < \|b\|$